SOLUTIONS OF PROBLEMS.

ALGEBRA.

403. Proposed by C. N. SCHMALL, New York City.

A torpedo-boat 40 miles from shore strikes a rock, making a rent in her hull which admits water at the rate of 15 tons in 48 minutes. The ship's pumps can expel 12 tons in an hour. If 60 tons of water is sufficient to sink the boat, find the average rate of steaming so that it may reach the shore just as it is about to sink.

Solution by Christian Hornung, Tiffin, O.

If x represents the rate per hour of steaming, then $40/x$ is the number of hours it takes to reach the shore, and in that time $15 \cdot 60 \cdot 40/48x$ tons of water will have entered the hull, and $12 \cdot 40/x$ tons will have been expelled. Hence $(15 \cdot 60/48) \cdot (40/x) - (12/1) \cdot (40/x) = 60$, and $x = 4\frac{1}{2}$.

Also solved by Emma Gibson, F. M. Morgan, Horace Olson, Clifford N. Mills, Walter C. Ells, and J. W. Clawson.

404. Proposed by V. M. SPUNAR, Chicago, Illinois.

Show that

$$(a+b)(a+b-1)\cdots(a+b-n+1) = a(a-1)(a-2)\cdots(a-n+1)$$
$$+\binom{n}{1}a(a-1)(a-2)\cdots(a-n+2)b+\binom{n}{2}a(a-1)(a-2)$$
$$\cdots(a-n+3)b(b-1)+\cdots+b(b-1)(b-2)\cdots(b-n+1).$$

Solution by A. M. Harding, University of Arkansas.

We have

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \cdots;$$

also

$$(1+x)^b = 1 + bx + \frac{b(b-1)}{2!}x^2 + \frac{b(b-1)(b-2)}{3!}x^3 + \cdots.$$

Multiply and equate the coefficients of x^n. Then

$$\binom{a+b}{n} = \binom{a}{n} + \binom{a}{n-1}\binom{b}{1} + \binom{a}{n-2}\binom{b}{2} + \cdots + \binom{b}{n}.$$

If we multiply both members of this equation by n! we obtain the desired result.

Also solved by Elijah Swift, who proved the proposition by induction.

405. Proposed by E. J. MOULTON, Northwestern University.

Given the alternating series